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ON NADAI'S ANALOGY FOR ELASTIC - PLASTIC TORSION

by

Philip G. Hodge, Jr.

Research

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Illinois Institute of Technology
Department of Mechanics
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ABSTRACT

Nadai's soap-film sand-hill analogy for torsion of an elastic-plastic cylinder has been used for both experimental and numerical determination of stresses despite the lack of a clear understanding of its limitations. A proof is given here that the analogy is always valid for a solid bar. It is shown by a counter example that the analogy is not generally valid for a hollow bar.

INTRODUCTION

Beginning with St. Venant's paper in 1856 [1], [3] torsion of prismatic and cylindrical bars has been the subject of intensive investigation. In 1903, Prandtl [2] showed that the problem was mathematically equivalent to the deflection of a membrane, an analogy which is useful not only for experimental determination of the stress distribution but also as a visual model to aid intuition. Nadai [3] showed that for a perfectly plastic material a corresponding analogy based on the shape of a sand-hill could be used for solid cylinders. The sand-hill analogy was extended to hollow cylinders by Sadowsky [4] in 1941. Nadai [3] also showed that the torsion of a solid cylinder of an elastic/perfectly-plastic bar could be represented by a combined analogy in which the membrane was inflated under a restraining roof. In 1944 Shaw [5] extended the Nadai analogy to hollow cylinders and used it as a basis for a numerical solution.

As was first pointed out by Prager and Hodge [6] in 1951, the Nadai analogy and the elastic/plastic torsion problem are identical only if certain restrictions are imposed on the shape of the boundary. The purpose of the present paper is to show that these restrictions are automatically satisfied for solid cylinders with simply-connected cross sections, but are generally not met for all stages of loading of hollow cylinders with multiply-connected cross sections.

3. Numbers in square brackets refer to the list of references at the end of the paper.

SOLID CYLINDER

We consider a solid cylinder whose cross section is a simply connected region A bounded by a curve C. Under the assumptions of St. Venant [1], equilibrium is automatically satisfied if the only non-zero stresses are given in terms of a stress function ψ by

$$\sigma_{xz} = \partial\psi/\partial y \qquad \sigma_{yz} = -\partial\psi/\partial x \tag{1}$$

In a region which has always been elastic, the stress function must satisfy

$$\nabla^2\psi = -2G\theta \tag{2}$$

where θ is the angle of twist per unit length and G the shear modulus; in a plastic region, the yield condition

$$|\nabla\psi| = k \tag{3}$$

must be satisfied. Finally, ψ must satisfy the boundary condition

$$\psi = 0 \quad \text{on} \quad C \tag{4}$$

The preceding formulation does not provide for a region which has a history of plastic behavior followed by elastic behavior. Even though the torque is assumed to be monotonically applied, it is conceivable that such a region could exist. Therefore, it is necessary to make a more precise statement of the elastic-plastic boundary value problem as follows;

<u>Torsion problem.</u> Given $\theta=\theta(t)$ such that $\theta(0) = 0$, $\dot{\theta} \geq 0$, and θ is continuous in time, find the stress function $\psi = \psi(x, y, t)$ such that

ψ is continuously differentiable in A (5a)

$\psi = 0$ on C (5b)

$|\nabla\psi| \leq k$ in A (5c)

If $|\nabla\psi| < k$ then $\nabla^2\dot{\psi} = -2G\dot{\theta}$ (5d)

If $|\nabla\psi| = k$ and $\nabla\psi \cdot \nabla\dot{\psi} < 0$, then $\nabla^2\dot{\psi} = -2G\dot{\theta}$ (5e).

The statement of (5d) and (5e) in terms of rates accounts for the fact that at some previous time in its history ψ may not have been subject to equation (2).

For the Nadai analogy, we denote the deflection of the membrane by ϕ and choose units so that the over-pressure and tension are related by $p/T = 2G\theta$. The "roof function" $\bar{\phi}$ is first defined by

$$\bar{\phi} \text{ is continuous and piecewise continuously differentiable in A} \tag{6a}$$

$$\bar{\phi} = 0 \quad \text{on C} \tag{6b}$$

$$|\nabla \bar{\phi}| = k \quad \text{in A} \tag{6c}$$

The roof function can be easily determined for any section as described, for example, in [6]. With $\bar{\phi}$ known, the analogy problem is formulated as follows:

Analogy problem. Given $\theta(t)$ such that $\theta(0) = 0$, $\dot{\theta} \geq 0$, and θ is continuous in time, find $\phi = \phi(x, y, t)$ such that

$$\phi \text{ is continuously differentiable in A} \tag{7a}$$

$$\phi = 0 \quad \text{on C} \tag{7b}$$

$$\phi \leq \bar{\phi} \quad \text{in A} \tag{7c}$$

$$\text{If } \phi < \bar{\phi} \text{ then } \nabla^2\phi = -2G\theta \tag{7d}$$

$$\text{If } \phi = \bar{\phi} \text{ and } \dot{\phi} < 0, \text{ then } \nabla^2\phi = -2G\theta \tag{7e}$$

For the Nadai analogy to be valid for a given problem, a solution $\psi = \phi$ where ϕ satisfies equations (7) must satisfy all of equations (5). Since the last three requirements on ϕ and ψ appear to be different, the correctness of the analogy is not obvious. However, for a simply-connected cross section we can show that satisfaction of (7) guarantees satisfaction of (5).

To this end, we first prove in the Appendix that $\dot{\phi} \geq 0$ throughout A. In view of (7c) this implies that once $\phi = \bar{\phi}$ at any point, $\dot{\phi} = 0$ from then on. Therefore requirement (7e) is superfluous and requirement (7d) could equally well be expressed in rate form as

$$\text{If } \phi < \bar{\phi} \text{ then } \nabla^2\dot{\phi} = -2G\dot{\theta} \tag{7d'}$$

Comparison of (7) and (5) then reveals that the only differences between them will be resolved if we can show that $|\nabla \phi| < k$ whenever $\phi < \bar{\phi}$. That this is indeed the case is proved in the Appendix by showing that $|\nabla \phi|$ cannot have a relative maximum in any region where $\nabla^2 \phi = -2G\theta$. Therefore, $\psi = \phi$ is a solution of the problem of elastic-plastic torsion and it follows from the general uniqueness theorems for an elastic/perfectly plastic material that it is the unique solution.

With the validity of the Nadai analogy established, it is easy to see that if C" is the plastic portion of the boundary C, then the entire plastic region must lie withing the region bounded by C" and its normals. Indeed if $\phi = \bar{\phi}$ at a point P which is a distance d from C along the normal N to C, then $\bar{\phi} = kd$ at P. Thus the average slope from P to the boundary along N is equal to k and since the magnitude of the gradient equals or exceeds the slope along a prescribed line, the average value of $|\nabla \phi|$ is equal or greater than k. Equations (7c) and (6c) then lead to the conclusion that $\phi = \bar{\phi}$ at each point on N.

HOLLOW CYLINDER

We consider a cylinder whose cross-section is the region A between an outer boundary C and an inner boundary C_i; the area enclosed by C_i is denoted by A_i.[4] The stress function must have a constant value ψ_i on C_i and the value of ψ_i is determined by the requirement that the displacements be single valued around the hole.

If the section is fully elastic, the equation for finding ψ_i is

$$\oint_{C_i} (\partial \psi / \partial n)\, ds = -2G\theta A_i \tag{8a}$$

If the section is partially plastic but there exists a curve $C_i{}'$ surrounding the hole which is and has always been elastic, then equation (8a) must be replaced by

$$\oint_{C_i{}'} (\partial \psi / \partial n) ds = -2G\theta A_i{}' \tag{8b}$$

where $A_i{}'$ is the total area bounded by $C_i{}'$. On the other hand, if there exists an elastic curve surrounding the hole but part of any such curve has been previously in a plastic state, the condition must be written in terms of rates:

$$\oint_{C_i{}'} (\partial \dot{\psi} / \partial n) ds = -2G\dot{\theta} A_i{}' \tag{8c}$$

Finally, if a single plastic region extends from the hole to the outer boundary, then ψ_i is obviously given by

$$\psi_i = kd \tag{8d}$$

where d is the shortest distance from the hole to the outer boundary along a normal to the outer boundary.

In the membrane analogy, that portion of the membrane which is over the hole is replaced by a rigid, weightless, horizontal disk, free to rise or fall. Equilibrium of the disk shows that ϕ must satisfy equation (8a).

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4. For simplicity of exposition we consider only a single hole; the extension to more holes is obvious.

For the sand-hill analogy we can write

$$\bar{\phi}_i = kd \tag{9}$$

analogus to (8d) and so construct a roof function. However, as Shaw [5] has shown by an example, the roof function defined by (9) may not be appropriate since $|\nabla \phi| = k$ may occur at the edge of the hole for $\phi < \bar{\phi}$. Rather, it is necessary to postulate a double roof attached to the weightless disk and rising or falling with it. The double roof is required by the fact that $\partial\phi/\partial n$ at the hole boundary may be positive or negative.

With the addition of the movable double roof, the analogy has reached a complexity where it is of questionable value for direct experimentation, but it is still helpful for forming an intuitive picture. However, it must be used with caution since it does not necessarily solve the torsion problem. To see this in an example, we consider the hollow filleted rectangle discussed in [5]. The insert in Figure 1 shows the cross section, and the detail shows part of the stress function for various values of θ. The bottom section is the flat membrane under zero torque. Next is shown the membrane at the maximum torque for which the cylinder is fully elastic and above this is a partially plastic stage. Data for these figures is taken from [5].

The top section shows the fully plastic roof. For the limiting case of fully plastic behavior $\phi = \bar{\phi}$ rises to a value above $\bar{\phi}_i$ near the corner and hence for almost fully plastic behavior the membrane is restrained by the upper roof. On the other hand, the section below shows that at this particular partially plastic state the membrane is restrained by the lower roof. Evidently there must be some stage intermediate to these two in which the membrane is not in contact with either roof. In this case, the analogy is not applicable, since requirement (7d) on ϕ does not correspond to requirement (5d) on ψ unless equation (2) has held for all previous times.

In view of the possible non-applicability of the Nadai analogy to multiply-connected cross sections, care must be taken in numerical or analytical solutions that one is truly solving the torsion problem rather than the analogy. As θ increases, the two solutions will agree so long as $\dot{\phi} = 0$ wherever $\phi = \bar{\phi}$ and $|\nabla\phi| < k$ where $\phi < \bar{\phi}$. When the analogy predicts that ϕ decreases numerically from $\bar{\phi}$, the torsion problem must be treated as a new analogy problem superposed on the original one at the instant the "unloading" begins. Since unloading may take place continuously in part of the section while loading continues in other parts, the resulting problem will be complicated by this phenomenon.

The existence of unloading under an increasing angle of twist has implications with regard to plasticity theory. In the first place, it is obvious that any "deformation law" will be quite inappropriate here. Further, the smooth transition from elastic to plastic behavior which was used in [7] for a solid section will lose much of its appeal since the transition between plastic flow and unloading is not smooth.

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APPENDIX

Proof that $\dot{\phi} \geq 0$. We note first that $\dot{\phi}$ is zero everywhere on C so that if it is anywhere negative it must have a relative minimum in the interior of A. At such a postulated minimum necessary conditions

$$\partial\dot{\phi}/\partial x = 0 \qquad \partial\dot{\phi}/\partial y = 0 \tag{10}$$

$$\partial^2\dot{\phi}/\partial x^2 \geq 0 \qquad \partial^2\dot{\phi}/\partial y^2 \geq 0 \tag{11}$$

Since $\dot{\phi}$ is continuously differentiable, equations (10) are obvious. Although the second derivatives may not exist along a curve which separates a roof region $\phi = \bar{\phi}$ from a free region $\phi < \bar{\phi}$, equations (11) are still necessary if the derivatives in question are interpreted as one-sided derivatives extending into either the roof or free region.

Evidently (11) are incompatible with $\nabla^2\dot{\phi} \leq 0$ and we shall show that this latter condition holds throughout. Let t_o be a time up to which $\phi = \bar{\phi}$ at a point implies $\dot{\phi} = 0$ (note that $t = 0$ is such a time so that some t_o exists) and let t_1 be any later time. Let $\bar{\theta}(x, y)$ denote the smallest angle of twist for which $\phi(x, y) = \bar{\phi}(x, y)$. Finally, let the change in ϕ from t_o to t_1 be denoted by $\nabla\phi = \phi_1 - \phi_o$.

Four possibilities exist depending upon whether $\phi = \bar{\phi}$ or not at t_o and t_1, but in each case $\nabla^2(\Delta\phi) \leq 0$. Thus:

$$\text{if } \phi_o < \bar{\phi} \text{ and } \phi_1 < \bar{\phi}, \text{ then } \nabla^2(\Delta\phi) = -2G(\theta_1 - \theta_o) < 0 \tag{12a}$$

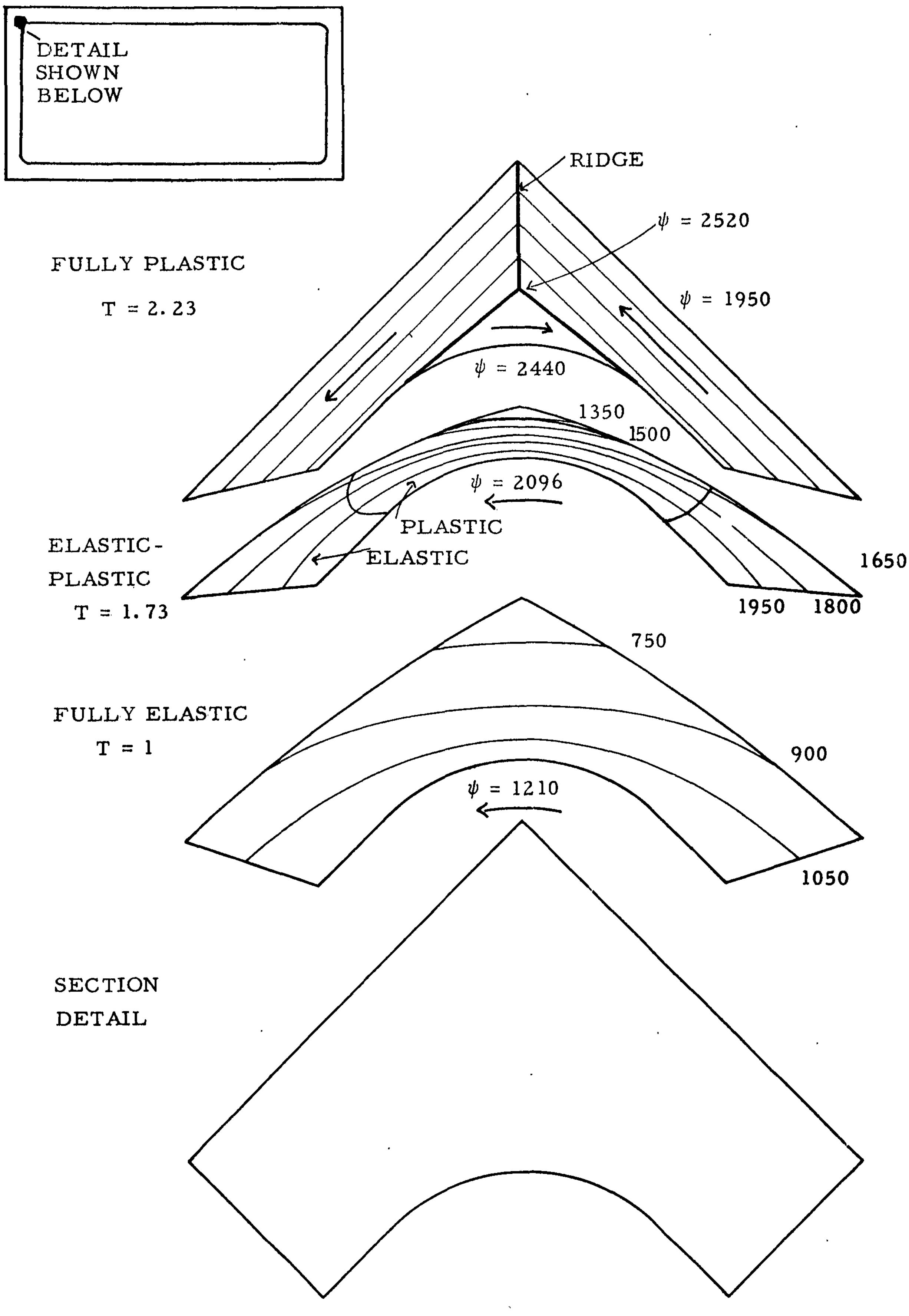
$$\text{if } \phi_o < \bar{\phi} \text{ and } \phi_1 = \bar{\phi}, \text{ then } \nabla^2(\Delta\phi) = -2G(\bar{\theta} - \theta_o) < 0 \tag{12b}$$

$$\text{if } \phi_o = \bar{\phi} \text{ and } \phi_1 < \bar{\phi}, \text{ then } \nabla^2(\Delta\phi) = -2G(\theta_1 - \bar{\theta}) < 0 \tag{12c}$$

$$\text{if } \phi_o = \bar{\phi} \text{ and } \phi_1 = \bar{\phi}, \text{ then } \nabla^2(\Delta\phi) = -2G(\bar{\theta} - \bar{\theta}) = 0 \tag{12d}$$

Therefore, starting from t_o any $\Delta\phi \geq 0$ and hence $\dot{\phi}(t_o) \geq 0$. However, since ϕ can never exceed $\bar{\phi}$, this conclusion renders case (12c) above impossible and leads to the conclusion $\dot{\phi} = 0$ if $\phi = \bar{\phi}$. This in turn removes the restriction on t_o so that the desired conclusion $\dot{\phi} \geq 0$ holds throughout A at all times.

<u>Proof that $|\nabla\phi| \leq k$.</u> Let A' denote that part of A where $\phi < \overline{\phi}$ and hence $\nabla^2 \phi = -2G\theta$. It is well known (see for instance [8]) that $|\nabla \phi|$ achieves its maximum only on the boundary B of A'. But B can consist only of parts C' of the original boundary C and the boundary B' between A' and the rest of A. On B', $|\nabla \phi| = k$ and on C' it follows from the definition of $\overline{\phi}$ that $|\nabla \phi| < k$. Therefore, the maximum value of $|\nabla\phi|$ on B is not greater than k and hence $|\nabla\phi| < k$ in the interior of A'.



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